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the locus required. Equation (15) may be factored by the solution of Algebra problem 250, the relevant factor, with subscripts omitted, being

$$\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)^2=\left(\frac{a^2+b^2}{a^2-b^2}\right)^2\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right).$$

Also solved by G. B. M. Zerr.

281. Proposed by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

In the proposition in solid geometry "If a line is perpendicular to each of two intersecting lines it is perpendicular to the plane of the lines," it is assumed that two intersecting lines have a common perpendicular. Prove it.

Solution by DR. GEORGE BRUCE HALSTED, Gambier, Ohio.

To prove that two intersecting straights have a common perpendicular.

To each of the given straights, a, b, at their intersection point A construct a perpendicular plane (Halsted's *Rational Geometry*, §337). They have a straight in common (H. R. G. I 6), which is perpendicular to a and perpendicular to b by H. R. G. §333, and that without *assuming* that any two intersecting straights have a common perpendicular.

Also solved by A. H. Holmes, Rev. J. H. Meyer, and G. B. M. Zerr.

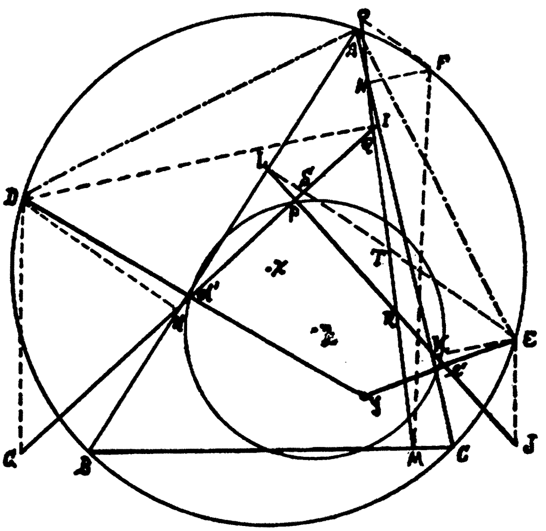
282. Proposed by REV. ALAN S. HAWKESWORTH, Allegheny, Pa.

The pedal lines of any two points on the circum-circle of a triangle concur in an angle equal to that subtended by the said points.

Solution by the PROPOSER.

Upon the circum-circle of triangle ABC take, first, any diameter DE. Then its pedal lines GHI and JKL will concur at P in a right angle.

Join DA, EA, and let GHI cut the perpendicular EL to AB in S. Then EKA and ELA being right angles, $EKLA$ are concyclic about EA; and angle $EAK=ELK$. Similarly, $DHIA$ are concyclic about DA; and $ADI=AHI$. Therefore the right angled triangles AID and SLH are similar; with angles DAI and LSP equal. And hence, even as the angles DAI and EAK are complementary, being within the semi-circle DAE; so also are their equals SLP and LSP. So that LPS is a right angle.



Next, take any other point F upon the circumcircle; and let its pedal line MNO meet those of D and E in Q and R, respectively. Join AF; and let MNO cut the perpendicular EL to AB in T, even as GHI cut it in S.

Then angle DAI has been shown equal to LSP; and hence also to QST. While EL and FO being parallel, QTS is equal to QOF. But $FNAO$ being concyclic about ON, QOF and FAN are equal; while angle DAF is compounded of DAI and FAN. So that GQO, or $QST+QTS$ are equal to DAF; and thus their supplement GQM to the angle subtended by arc DAF.

And lastly; since PRQ is the complement to PQR, even as the angle subtended by arc FE is to that subtended by arc DF; the said angles PRQ and that subtended by arc FE must be also equal. And similarly for any point upon the circum-circle.

Corollary. DE being a diameter of the circum-circle, the medial points,—say u, v, and w,—of the sides AB, BC, and CA of the inscribed triangle must also bisect the segments LH, GJ, and KI cut off upon said sides by its pedal lines GHI and JKL. Each bisecting perpendicular,—say xu, xv, and xw,—from x, the circum-center, being parallel to DH and EL, DG and EJ, and DI and EK, respectively. And thus the said medial points u, v, and w are always the circum-centers of the right angled triangles HPL, GPJ, and KPI, respectively.

Also solved by G. W. Greenwood, and J. Scheffer.

283. Proposed by **REV. ALAN S. HAWKESWORTH**, Allegheny, Pa.

The right angled intersection of the pedal lines of any diameter of the circumcircle lies on the "nine points circle" of the inscribed triangle.

Solution by **J. SCHEFFER, A. M.**, Hagerstown, Md., and the **PROPOSER**.

Let x [see figure above] be the circum-center, y the ortho-center, and z the center of the nine points circle of triangle ABC; passing, by construction, through the feet of the perpendiculars from the ortho-center upon the sides of the triangle, and also through the medial points of the lines joining the ortho-center to the vertices of the triangle.

Then, by known theorems, the center of z of the nine points circle bisects xy, the distance between the ortho- and circum-centers. While the said lines from the orth-ocenter perpendicular to the sides of the triangle cut its circum-circle at twice their distance to these sides. So that the ortho-center Y is evidently also a center of similitude to the nine points and circum-circles; with all magnitudes of the latter half those of the former.

And hence d', where DY, by a known theorem, is bisected by the pedal line GHI of D, is also its intersection by the nine points circle. And similarly, e', the middle point of YE, lies both on the pedal line JKL of E, and on the nine points circle.

Therefore $d'e'$ is parallel to DE, and bisects XY in Z, and is thus a diameter of the nine points circle; even as DXE is of the circum-circle. So that $d'Pe'$ being a right angle, P lies on the nine points circle.